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Temperature gradient-induced instability in a solid state plasma

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Abstract. A solid state plasma subject to crossed electric and magnetic fields and with electron drift is found to be unstable when temperature and density gradients are present in the same direction as the electron drift velocity. The frequency of the oscillations and their maximum growth rate are plotted as a function of the magnetic field for a sample of n-type indium antimonide.

When a current is passed through a sample of semiconducting material the temperature is raised by ohmic heating. If one end of the sample is maintained at a constant temperature, a temperature gradient will be set up along the direction of the current. In a semiconducting material below room temperature the density of electrons and holes increases with increasing temperature so that a density gradient is also set up. Application of a magnetic field perpendicular to the current and the gradients produces a transverse drift. Instabilities in both gaseous (Tsai *et al* 1970) and solid state (Suzuki 1966) plasmas due to this transverse drift have been observed.

Work by A R Taylor (1970, private communication) appears to indicate that voltage fluctuations between the ends of a rod of semiconducting material are obtained when a longitudinal temperature gradient is applied in the presence of a steady longitudinal electric field and a steady magnetic field with a transverse component as illustrated in figure 1. The amplitude of the oscillations was found to be a maximum when the magnetic field was perpendicular to the temperature gradient. In this experiment the

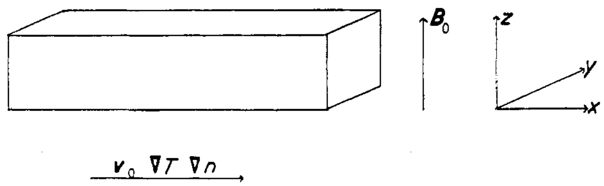

Figure 1. Geometry of semiconductor sample.

transverse drift produced a transverse density gradient which was considered negligible compared with the longitudinal density gradient. In this note it is shown that longitudinal temperature and density gradients can cause unstable behaviour in semiconducting media.

The following assumptions have been made: (i) the waves are electrostatic and low frequency; (ii) wavelengths are much smaller than the density and temperature decay lengths; (iii) inertia terms in the equation of motion can be neglected; (iv) the material is an n-type semiconductor in which the number of electrons is much greater than the number of holes and in this work the effect of holes has not been taken into account.

The calculation is carried out for two cases (*a*) isothermal electrons for which analytic results are obtained and (*b*) nonisothermal electrons for which numerical results are presented.

The equations of continuity, momentum and heat balance for the electrons are (Braginskii 1965)

$$\frac{dn}{dt}+n\nabla . v = 0 \tag{1}$$

$$0 = -\frac{\nabla p}{nm}-\frac{e}{m}\left(E+\frac{1}{c}v\times B\right)+R \tag{2}$$

$$\frac{3}{2}n\frac{dT}{dt}-T\frac{dn}{dt}+\nabla . q = 0 \tag{3}$$

where $R = R_u+R_T$. The frictional force $R_u = -v\tau^{-1}$ is due to electron collisions with the lattice while the component R_T is due to the gradient in the electron temperature and is given by

$$R_T = -\frac{1}{m}\nabla_{\parallel}T-\frac{1}{m}\frac{1}{1+\Omega^2\tau^2}\nabla_{\perp}T-\frac{1}{m}\frac{\Omega\tau}{1+\Omega^2\tau^2}\hat{b}\times\nabla_{\perp}T$$

where τ is the electron–lattice collision time, Ω the electron cyclotron frequency and $\hat{b}$ is a unit vector in the direction of the magnetic field. The symbols $\parallel$, $\perp$ refer to components parallel and perpendicular to the magnetic field. In (3) $q = q_u+q_T$ with

$$q_u = nTv_{\parallel}+nT(1+\Omega^2\tau^2)^{-1}v_{\perp}+nT\Omega\tau(1+\Omega^2\tau^2)^{-1}\hat{b}\times v_{\perp}$$

$$q_T = -nT\frac{\tau}{m}\nabla_{\parallel}T-\frac{\tau}{m}nT(1+\Omega^2\tau^2)^{-1}\nabla_{\perp}T-\frac{nT}{m}\Omega\tau^2(1+\Omega^2\tau^2)^{-1}\hat{b}\times\nabla_{\perp}T$$

that is, q_u is the heat flux due to the motion of the electrons through the lattice and q_T is the heat flux due to the gradient in electron temperature.

To obtain a dispersion relation for the electrons the perturbed currents are calculated in terms of the perturbed electric fields from which the dielectric tensor ϵ_{ij} can be found. Using the electrostatic dispersion relation $n_i\epsilon_{ij}n_j = 0$ with $n_i = ck_i\omega^{-1}$ the electron dispersion relation may be written as an equation for the complex frequency ω. If one restricts the discussion to the case of isothermal electrons and ignores temperature perturbations analytic results may be found. A quadratic equation for ω is obtained giving two modes of propagation. One of these is a drift wave moving with phase velocity equal to the electron drift velocity. This wave is subject to collisional damping and for most solid state plasmas the damping is found to be much greater than the growth due to the density gradient.

The real and imaginary parts of the frequency of the second mode are given by

$$\omega_R = \frac{S_1v_0\Omega\omega_p^{-2}(k_xv_0\Omega\tau-k_yv_0)\{2+(\Omega^2\tau^2k_z^2/k^2)-\Omega\tau k_xk_y/k^2\}}{\{1+(\Omega^2\tau^2k_z^2/k^2)\}^2+\Omega^2\omega_p^{-4}(k_xv_0\Omega\tau-k_yv_0)^2} \tag{4}$$

$$\omega_I = \frac{S_1v_0\{1+(\Omega^2\tau^2k_z^2/k^2)\}\{2+(\Omega^2\tau^2k_z^2/k^2)-\Omega\tau k_xk_y/k^2\}}{\{1+(\Omega^2\tau^2k_z^2/k^2)\}^2+\Omega^2\omega_p^{-4}(k_xv_0\Omega\tau-k_yv_0)^2} \tag{5}$$

where v_0 is the electron drift velocity, ω_p the electron plasma frequency and $\boldsymbol{k}$ the wavevector while $S_1 = (1/n)(\partial n/\partial x)$. When $S_1 v_0 > 0$, that is, when the electron drift velocity is in the direction of increasing density the wave has a positive growth rate for all values of the magnetic field. The imaginary part of the frequency has a minimum value when

$$k_z = 0 \qquad k_y = k_x$$

a maximum value when

$$k_z = 0 \qquad k_y = -k_x$$

and it approaches a constant value $2S_1v_0$ for large values of $|k_y|$. In figure 2 the imaginary part of the frequency is plotted against k_y, the wavenumber perpendicular to

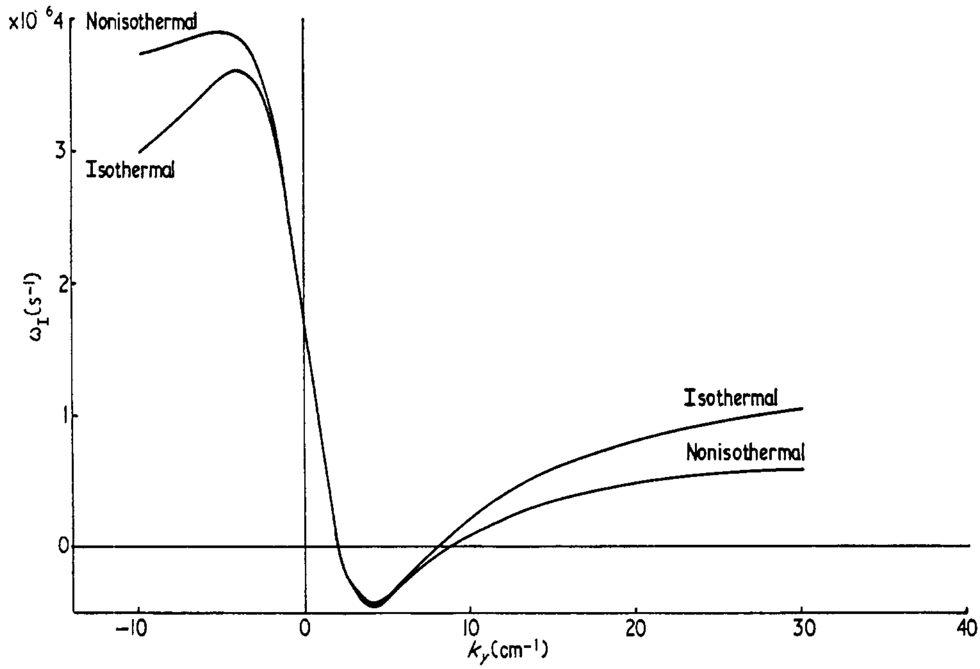

Figure 2. Graphs of imaginary frequency ω_I, against k_y the wavenumber perpendicular to both the drift velocity and the magnetic field for an unstable wave in n-InSb. $B_0 = 400$ G, $k_x = 4$, $k_z = 0$.

both the electron drift velocity and the magnetic field. The maximum growth rate is given by

$$\omega_I = \frac{S_1 v_0}{2}(4+\Omega\tau)\{1+\Omega^2 k_x^2 v_0^2 \omega_p^{-4}(1+\Omega\tau)^2\}^{-1}.$$

This is directly proportional to the density gradient and to the electron drift velocity. For values of $\Omega\tau \leqslant 100$ this is approximately a linear function of the magnetic field but it reaches a maximum value when $\Omega = 3^{1/4}\omega_p(k_x v_0 \tau)^{-1/2}$. The corresponding frequency is given by

$$\omega_R = k_x S_1 v_0^2 (2\omega_p^2)^{-1}\Omega(1+\Omega\tau)(4+\Omega\tau)$$

that is, directly proportional to the density gradient and to the square of the electron drift velocity. The maximum growth rate and the corresponding frequency are plotted as a function of the magnetic field in figure 3. Both figures have been plotted for the

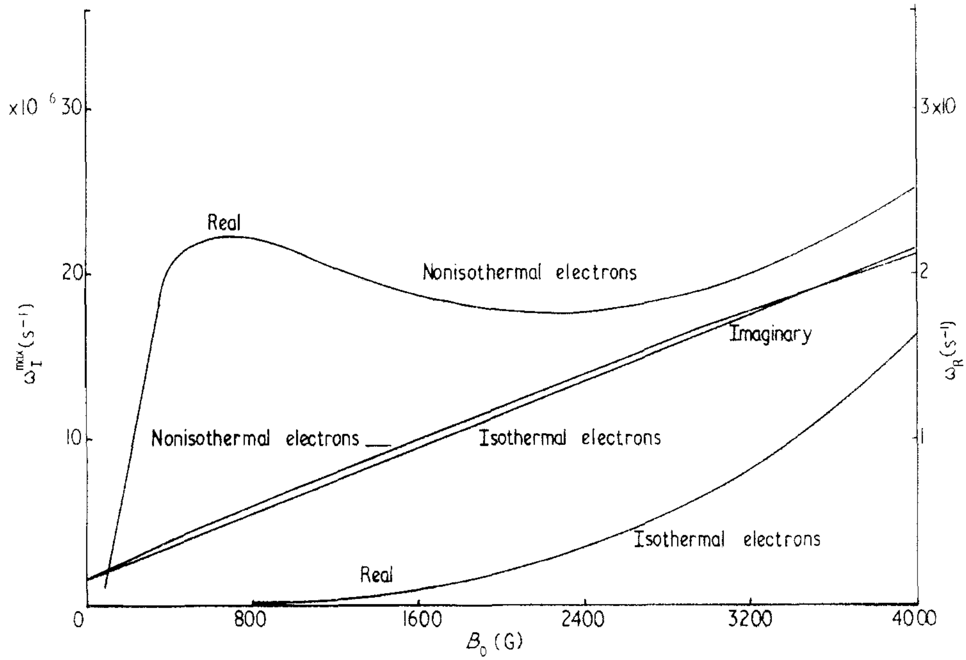

Figure 3. Graphs of maximum growth rate ω_I^{max} and real frequency ω_R against magnetic field B_0 for an unstable wave in n-InSb.

following parameters for indium antimonide (Smith 1959):

$$T = 100\,\mathrm{K} \qquad n = 5\times10^{11}\,\mathrm{cm}^{-3} \qquad v_0 = 10^6\,\mathrm{cm\,s}^{-1}$$

$$\tau = 10^{-11}\,\mathrm{s}$$

$$\frac{1}{n}\frac{\partial n}{\partial x} = 0{\cdot}8 \qquad \frac{1}{T}\frac{\partial T}{\partial x} = 0{\cdot}5 \qquad k_z = 0 \qquad k_x = 4\,\mathrm{cm}^{-1}.$$

When the restriction of isothermal electrons is removed and temperature perturbations are included the dispersion relation must be solved numerically. Again the growth rate is shown to be positive for all values of the magnetic field. It has a minimum value when

$$k_z = 0 \qquad k_y = k_x$$

and a maximum value when

$$k_z = 0 \qquad k_y \leqslant -k_x.$$

In figure 2 the imaginary part of the frequency is compared for isothermal and nonisothermal cases. In figure 3 the maximum growth rates for isothermal and nonisothermal electrons are compared and one observes that for values of the magnetic field up to 3500 G the maximum growth rate is slightly increased for nonisothermal electrons. Figure 3 illustrates how, for magnetic fields below 5000 G, the real part of the frequency is radically altered by the assumption of nonisothermal electrons. Within this range it is found to be much less than the cyclotron frequency and is in fact close to the frequency of the oscillations observed by Taylor (1970, private communication).

When a small number of holes is included it is found that the growth rate decreases while the frequency increases. A discussion of the effect of holes on this instability will be included in a later paper.

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